## Advanced quantum information: entanglement and nonlocality

### 3. Homework sheet

Solutions to be submitted via email to a.streltsov@cent.uw.edu.pl
Please submit a single pdf file using "Solutions advanced quantum information" in the subject line. Latest date for submission: 29. April 2020

**Problem 1 (Lecture notes Section 6)**

The negativity of a quantum state $\rho^{AB}$ is defined as

$$E_n(\rho^{AB}) = \frac{||\rho^{T_B}||_1 - 1}{2},$$

where $\rho^{T_B}$ denotes partial transpose, and $||M||_1 = \mathrm{Tr}\sqrt{M^\dagger M}$ is the trace norm of the matrix $M$.

**a)** Prove that negativity fulfills

$$E_n(\rho^{AB}) \geq 0$$

with $E_n(\rho^{AB}) = 0$ when $\rho^{AB}$ is separable.

**b)** Using the properties of trace norm (see page 32 of Lecture notes) prove that negativity is convex:

$$E_n\left(\sum_i p_i \rho_i^{AB}\right) \leq \sum_i p_i E_n\left(\rho_i^{AB}\right).$$

**Problem 2 (Lecture notes Section 6)**

For a state of two qubits the concurrence $C$ is given as

$$C(\rho^{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},$$

where $\lambda_i$ are the square roots (in decreasing order) of the eigenvalues of the non-Hermitian matrix $\rho\tilde{\rho}$, with

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y),$$

the Pauli matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\rho^*$ denotes entry-wise complex conjugation. The entanglement of formation of the state can then be evaluated as

$$E_f(\rho^{AB}) = h\left(\frac{1 + \sqrt{1 - C^2(\rho^{AB})}}{2}\right)$$

with the binary entropy $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$.

Consider now a two-qubit state of the form

$$\rho^{AB} = p_1 |\Psi^+\rangle\langle\Psi^+| + p_2 |\Psi^-\rangle\langle\Psi^-| + p_3 |\Phi^+\rangle\langle\Phi^+| + p_4 |\Phi^-\rangle\langle\Phi^-|$$

with $p_i \geq 0$ and $\sum_i p_i = 1$.

**a)** Evaluate the concurrence, entanglement of formation, and negativity of $\rho^{AB}$ as a function of the probabilities $p_i$.

**b)** Use the previous results to evaluate the concurrence, entanglement of formation, and negativity for the two-qubit Werner state

$$\rho^{AB} = p |\Psi^-\rangle\langle\Psi^-| + (1-p)\frac{\mathbb{1}_4}{4}$$

as a function of $p$. Plot the concurrence, entanglement of formation, and negativity for $0 \leq p \leq 1$.