Advanced quantum information: entanglement and nonlocality

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7th class
April 22, 2020

Advanced quantum information (7th class)

- Howework and lecture notes:
  http://qot.cent.uw.edu.pl/teaching/
- Comments on 2. homework sheet have been sent on 21. April

- These slides available online now

- If you have a question during the class:
  - unmute audio first
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Outline

1. Entanglement quantification
   - Distance-based entanglement measures
   - Distillable entanglement and entanglement cost
2. Entanglement monogamy
3. Applications of entanglement
   - Superdense coding
   - Quantum state merging
   - Quantum computing

Outline

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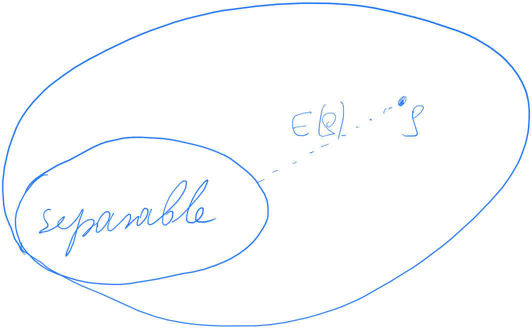
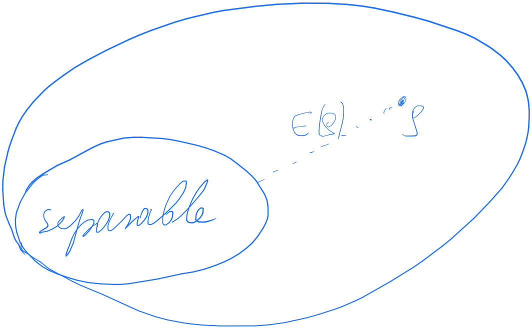
Distance-based entanglement measures



Distance-based entanglement measures

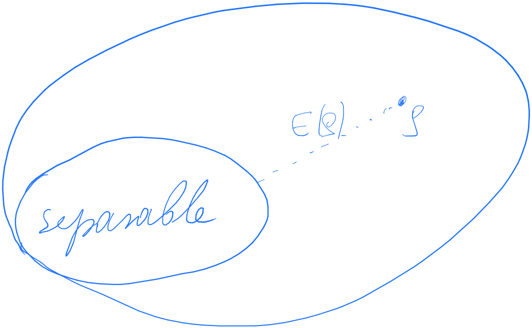


For a distance function $D(\rho, \sigma)$ define

$$E(\rho) = \inf_{\sigma \in \mathcal{S}} D(\rho, \sigma)$$

with infimum over separable states $\mathcal{S}$

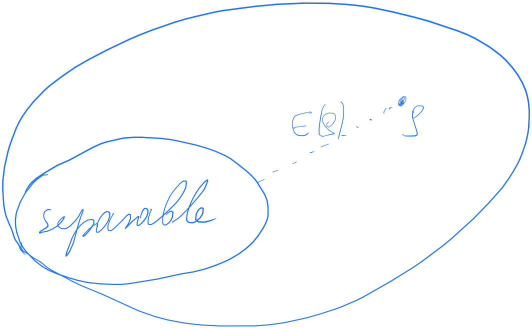
Distance-based entanglement measures



E is an entanglement measure if:

1. $D(\rho, \sigma) \geq 0$ with equality for $\rho = \sigma$

Distance-based entanglement measures



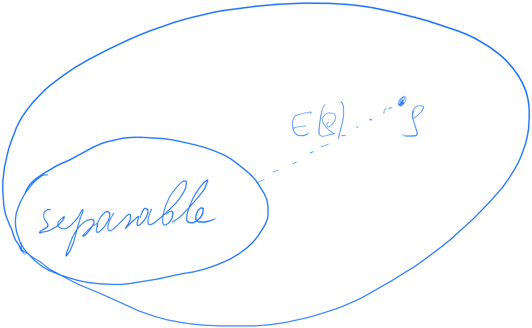
E is an entanglement measure if:

1. $D(\rho, \sigma) \geq 0$ with equality for $\rho = \sigma$
2. D is contractive under quantum operations:

$$D(\Lambda[\rho], \Lambda[\sigma]) \leq D(\rho, \sigma)$$

for any quantum operation Λ

Distance-based entanglement measures



Exercise: prove that

$$E(\rho) = \inf_{\sigma \in \mathcal{S}} D(\rho, \sigma)$$

is nonnegative and zero on separable states for any D with the property $D(\rho, \sigma) \geq 0$ and $D(\rho, \rho) = 0$

Distance-based entanglement measures

Proof that $E(\rho) = \inf_{\sigma \in \mathcal{S}} D(\rho, \sigma)$ does not increase under LOCC:

$$E(\Lambda_{\text{LOCC}}[\rho]) \leq E(\rho)$$

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- We have

$$\begin{aligned} E\left(\Lambda_{\rm LOCC}[\rho]\right) &= \min_{\mu \in \mathcal{S}} D\left(\Lambda_{\rm LOCC}[\rho], \mu\right) \leq D\left(\Lambda_{\rm LOCC}[\rho], \Lambda_{\rm LOCC}[\sigma]\right) \\ &\leq D(\rho, \sigma) = E(\rho) \end{aligned}$$

Q.E.D.

Distance-based entanglement measures

Examples for distances fulfilling $D(\Lambda[\rho], \Lambda[\sigma]) \leq D(\rho, \sigma)$:

Distance-based entanglement measures

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- Quantum relative entropy

$$S(\rho \| \sigma) = \mathrm{Tr}[\rho \log_2 \rho] - \mathrm{Tr}[\rho \log_2 \sigma]$$

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- Relative entropy of entanglement:

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- For mixed states: hard to compute in general

Distance-based entanglement measures

Examples for distances fulfilling $D(\Lambda[\rho], \Lambda[\sigma]) \leq D(\rho, \sigma)$:

- Bures distance

$$B(\rho, \sigma) = \sqrt{2 - 2F(\rho, \sigma)}$$

with fidelity $F(\rho, \sigma) = \mathrm{Tr}\, \sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}$

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- Trace distance

$$D_t(\rho, \sigma) = \frac{1}{2}||\rho - \sigma||_1$$

with the trace norm $||M||_1 = \mathrm{Tr}\, \sqrt{M^\dagger M}$

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- **Distillable entanglement:** singlet rate obtainable from a quantum state ρ via LOCC in the asymptotic limit

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Distillable entanglement and entanglement cost

- E_d and E_c are special cases of **asymptotic state-conversion rates**

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- For pure states holds

$$R(|\psi\rangle \to |\phi\rangle) = \frac{S(\rho_\psi)}{S(\rho_\phi)},$$

where ρ_ψ is the reduced state of $|\psi\rangle$

Distillable entanglement and entanglement cost

Bounds on E_d and E_c:

Distillable entanglement and entanglement cost

Bounds on E_d and E_c:

$$E_d(\rho^{AB}) \leq E_c(\rho^{AB}) \leq E_f(\rho^{AB})$$

Distillable entanglement and entanglement cost

Bounds on E_d and E_c:

$$E_r(\rho^{AB}) \geq E_d(\rho^{AB}) \geq \max\left\{S(\rho^{A}) - S(\rho^{AB}), S(\rho^{B}) - S(\rho^{AB})\right\}$$

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Application: consider maximally correlated state

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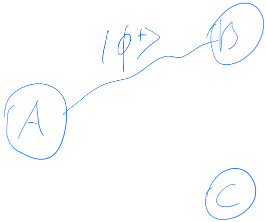
$$\Rightarrow E_d(\rho_{\mathrm{mc}}^{AB}) = S(\rho_{\mathrm{mc}}^{A}) - S(\rho_{\mathrm{mc}}^{AB})$$

Outline

Entanglement monogamy

Exercise: If two qubits A and B are in the state $|\Phi^+\rangle$, prove that they cannot be correlated with another qubit C



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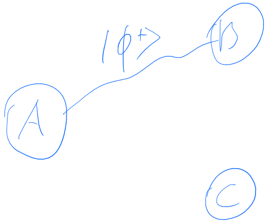

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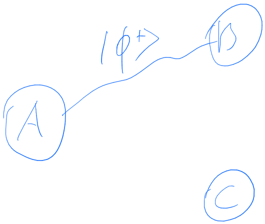

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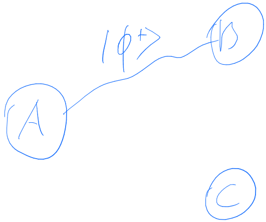


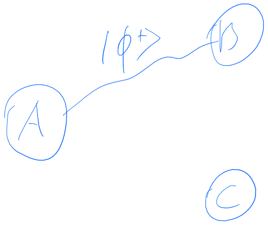
Solution:

- Consider total state ρ^{ABC}
- Reduced state is $\rho^{AB} = |\Phi^+\rangle\langle\Phi^+|^{AB}$
- $\Rightarrow$ total state must be

$$\rho^{ABC} = |\Phi^+\rangle\langle\Phi^+|^{AB} \otimes \rho^{C}$$

Entanglement monogamy

Entanglement monogamy: If two qubits *A* and *B* are maximally entangled, they cannot be entangled with another qubit *C*



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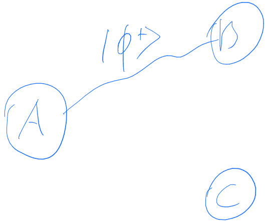

Compare to classical random variables: a classical random variable A can be maximally correlated with B and C at the same time:

$$\rho^{ABC} = \frac{1}{2}\,|000\rangle\langle 000| + \frac{1}{2}\,|111\rangle\langle 111|$$

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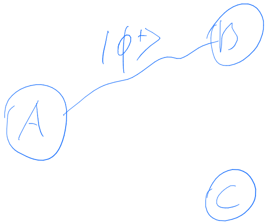

- For a pure state $|\psi\rangle^{ABC}$ it holds

$$C^2_{A:B} + C^2_{A:C} \leq C^2_{A:BC}$$

Entanglement monogamy

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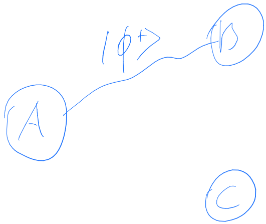

- For a pure state $|\psi\rangle^{ABC}$ it holds

$$C_{A:B}^2 + C_{A:C}^2 \leq C_{A:BC}^2$$

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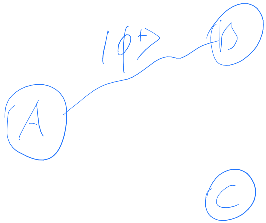

- For a pure state $|\psi\rangle^{ABC}$ it holds

$$C_{A:B}^2 + C_{A:C}^2 \leq C_{A:BC}^2$$

- $C_{A:B}$ and $C_{A:C}$: concurrence of the reduced state ρ^{AB} and ρ^{AC}
- $C_{A:BC} = \sqrt{2\left(1 - \mathrm{Tr}\left[(\rho^A)^2\right]\right)}$

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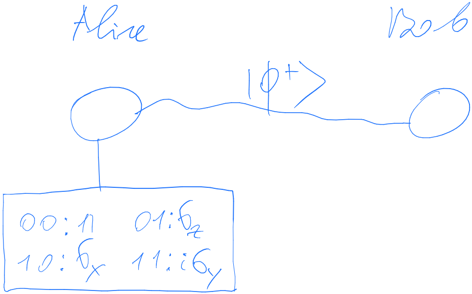
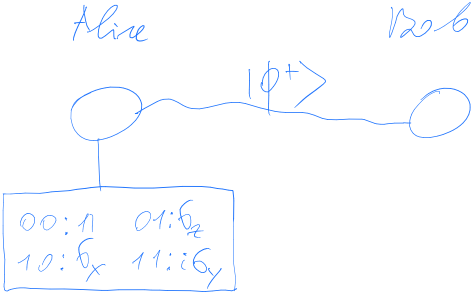
Superdense coding



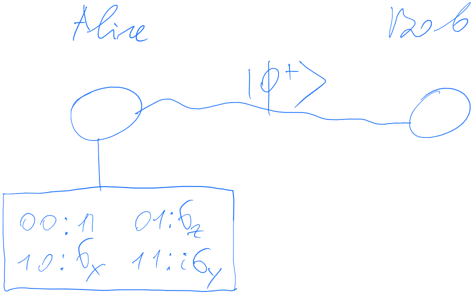
Alice can encode **two bits** of information into **one qubit**

Superdense coding



1. Alice applies a unitary on her qubit, depending on which two bits she wants to send to Bob

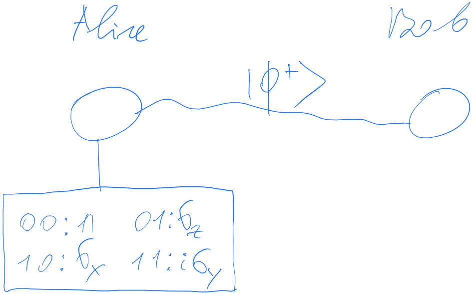
Superdense coding



Resulting states are

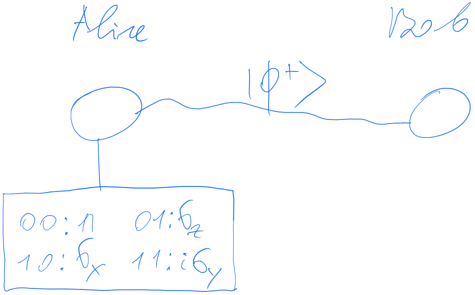
$$00 : |\Phi^+\rangle \to |\Phi^+\rangle \,, 01 : |\Phi^+\rangle \to |\Phi^-\rangle$$
$$10 : |\Phi^+\rangle \to |\Psi^+\rangle \,, 11 : |\Phi^+\rangle \to |\Psi^-\rangle$$

Superdense coding



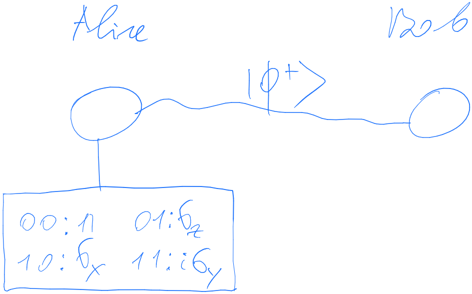
2. Alice sends her qubit to Bob, who is now in possession of one of the four Bell states

Superdense coding



3. Bob applies a von Neumann measurement in the maximally entangled basis. From his outcome, he can directly read off the two bits encoded by Alice.

Superdense coding



Two bits is the maximal amount of classical information that one qubit can carry

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Quantum state merging

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- Alice and Bob can apply local operations and classical communication, and also use additional singlets
- **Aim:** send Alice's part of the state to Bob without disturbing the state
- Asymptotic limit: many copies of $|\psi\rangle^{RAB}$

Quantum state merging

Alice and Bob need to share singlets at rate

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B})$$

Quantum state merging

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$$S(A|B) = S(\rho^{AB}) - S(\rho^{B})$$

- $S(A|B)$: **quantum conditional entropy**, can be positive or negative

Quantum state merging

Alice and Bob need to share singlets at rate

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B})$$

- $S(A|B) \geq 0$: Alice and Bob need singlets at rate $S(A|B)$

Quantum state merging

Alice and Bob need to share singlets at rate

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B})$$

- $S(A|B) \geq 0$: Alice and Bob need singlets at rate $S(A|B)$
- $S(A|B) < 0$: Alice and Bob can achieve merging without singlets, and additionally the process will gain singlets at rate $-S(A|B)$

Quantum state merging

Alice and Bob need to share singlets at rate

$$S(A|B) = S(\rho^{AB}) - S(\rho^{B})$$

Exercise: Assume that Bob is uncorrelated, $|\psi\rangle^{RAB} = |\psi\rangle^{RA} \otimes |\phi\rangle^{B}$. Evaluate $S(A|B)$.

Quantum state merging

Solution: For $|\psi\rangle^{RAB} = |\psi\rangle^{RA} \otimes |\phi\rangle^{B}$ we have $S(A|B) = S(\rho^{A})$

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Asymptotic version of quantum teleportation

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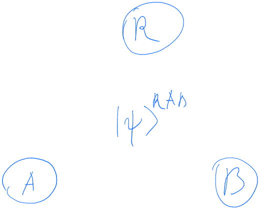
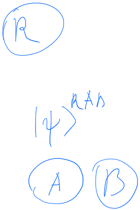
Asymptotic version of quantum teleportation

Equivalent to **Schumacher compression**:

- Alice compresses $n \gg 1$ copies of ρ^A into $nS(\rho^A)$ qubits

Quantum state merging





Solution: For $|\psi\rangle^{RAB} = |\psi\rangle^{RA} \otimes |\phi\rangle^{B}$ we have $S(A|B) = S(\rho^A)$

Asymptotic version of quantum teleportation

Equivalent to **Schumacher compression**:

- Alice compresses $n \gg 1$ copies of ρ^A into $nS(\rho^A)$ qubits
- Alice teleports these qubits by consuming $nS(\rho^A)$ singlets

Quantum state merging

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Asymptotic version of quantum teleportation

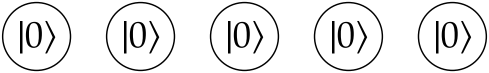
Equivalent to **Schumacher compression**:

- Alice compresses $n \gg 1$ copies of ρ^A into $nS(\rho^A)$ qubits
- Alice teleports these qubits by consuming $nS(\rho^A)$ singlets
- This compression scheme is optimal asymptotically

Outline

Entanglement in quantum computation

Scheme of a typical quantum algorithm:



Entanglement in quantum computation

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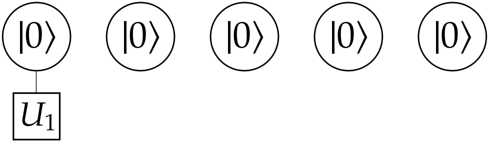

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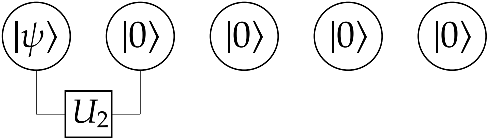

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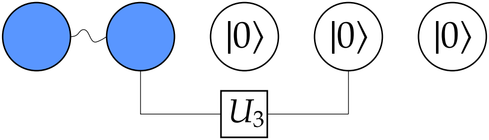

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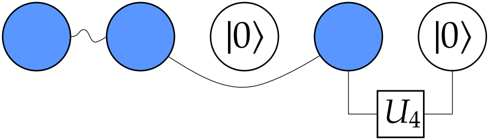

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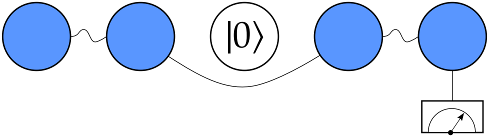

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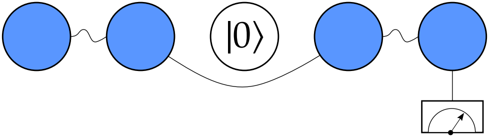

Examples of quantum algorithms:

Entanglement in quantum computation

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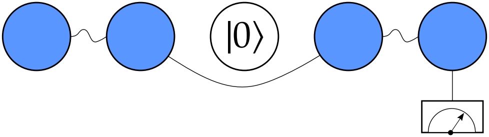

Examples of quantum algorithms:

- Shor's algorithm for efficient prime factorization

Entanglement in quantum computation

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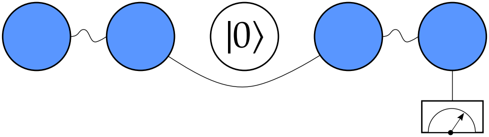

Examples of quantum algorithms:

- Shor's algorithm for efficient prime factorization
- Grover's algorithm for efficient database search

Entanglement in quantum computation

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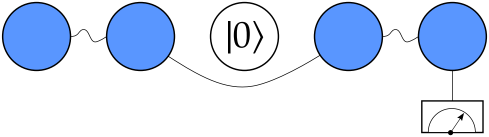

Pure state quantum computation:

- Entanglement required for an exponential speedup

Entanglement in quantum computation

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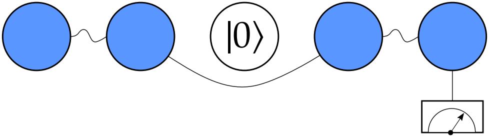

Pure state quantum computation:

- Entanglement required for an exponential speedup
- Arbitrary little (but nonzero) entanglement is enough

Entanglement in quantum computation

Scheme of a typical quantum algorithm:

Mixed state quantum computation:

- Very little is known

Entanglement in quantum computation

Scheme of a typical quantum algorithm:

Mixed state quantum computation:

- Very little is known
- Quantum speedup might be possible with separable states