Advanced quantum information: entanglement and nonlocality

3. Homework sheet

Solutions to be submitted via email to t.kondra@cent.uw.edu.pl
Please submit a single pdf file using "Solutions Advanced Quantum Information" in the subject line. Latest date for submission: 19. April 2022

Problem 1 (Lecture notes Section 6)

The negativity of a quantum state ρ^{AB} is defined as

$$E_n(\rho^{AB}) = \frac{\|\rho^{T_B}\|_1 - 1}{2},$$

where ρ^{T_B} denotes partial transpose, and $\|M\|_1 = \mathrm{Tr}\sqrt{M^\dagger M}$ is the trace norm of the matrix M.

a) Prove that negativity fulfills

$$E_n(\rho^{AB}) \geq 0$$

with $E_n(\rho^{AB}) = 0$ when ρ^{AB} is separable.

b) Using the properties of trace norm prove that negativity is convex:

$$E_n\left(\sum_i p_i \rho_i^{AB}\right) \leq \sum_i p_i E_n\left(\rho_i^{AB}\right).$$

Problem 2 (Lecture notes Section 6)

For a state of two qubits the concurrence C is given as

$$C(\rho^{AB}) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\},$$

where λ_i are the square roots (in decreasing order) of the eigenvalues of the non-Hermitian matrix $\rho\tilde{\rho}$, with

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y),$$

the Pauli matrix $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and ρ^* denotes entry-wise complex conjugation. The entanglement of formation of the state can then be evaluated as

$$E_f(\rho^{AB}) = h\left(\frac{1 + \sqrt{1 - C^2(\rho^{AB})}}{2}\right)$$

with the binary entropy $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$.

Consider now a two-qubit state of the form

$$\rho^{AB} = p_1 |\Psi^+\rangle\langle\Psi^+| + p_2 |\Psi^-\rangle\langle\Psi^-| + p_3 |\Phi^+\rangle\langle\Phi^+| + p_4 |\Phi^-\rangle\langle\Phi^-|$$

with $p_i \geq 0$ and $\sum_i p_i = 1$.

a) Evaluate the concurrence, entanglement of formation, and negativity of ρ^{AB} as a function of the probabilities p_i.

b) Use the previous results to evaluate the concurrence, entanglement of formation, and negativity for the two-qubit Werner state

$$\rho^{AB} = p |\Psi^-\rangle\langle\Psi^-| + (1-p)\frac{\mathbb{1}_4}{4}$$

as a function of p. Plot the concurrence, entanglement of formation, and negativity for $0 \leq p \leq 1$.